**Topics: Normal distribution, Functions of Random Variables**

1. The time required for servicing transmissions is normally distributed with *μ* = 45 minutes and *σ* = 8 minutes. The service manager plans to have work begin on the transmission of a customer’s car 10 minutes after the car is dropped off and the customer is told that the car will be ready within 1 hour from drop-off. What is the probability that the service manager cannot meet his commitment?
2. 0.3875
3. 0.2676
4. 0.5
5. 0.6987

Answer:

µ = 45 minutes

σ = 8 minutes

X = 45 – 10 = 50 minutes

Calculating z-score

= (X - µ) / σ

= (50-45)/8

= 0.625

Probability = 1-z

= 1-0.733

= 0.2676

1. The current age (in years) of 400 clerical employees at an insurance claims processing center is normally distributed with mean *μ* = 38 and Standard deviation *σ* =6. For each statement below, please specify True/False. If false, briefly explain why.
2. More employees at the processing center are older than 44 than between 38 and 44.

Answer:

Z score = (value –mean)/SD

Z score for 44 = (44-38)/6 =1 =84.13%

People above 44 =100 - 84.13 = 15.87 =63 out of 400

Z score for 38 (38-38) = 0 = 50 %

People between 38 and 44 age 84.13 – 50=34.13 =138 of 400

**More employees at the processing center are older than 44 than between 38 and 44 is False**

1. A training program for employees under the age of 30 at the center would be expected to attract about 36 employees.

Answer:

Z score for 30 = (30-38)/6 = -1.33 = 9.15% = 36 out of 400

**Hence A training program for employee under the age 30 at the center would be expected to attract about 36 employee - True**

1. If *X1*~ *N*(μ, σ2) and *X*2 ~ *N*(μ, σ2) are *iid*normal random variables, then what is the difference between 2 *X*1 and *X*1 + *X*2? Discuss both their distributions and parameters.

Answer:

According to the central limit theorem any large sum of independent identically distributed random variable is approximately Normal

The normal distribution is defined by two parameter the mean and variance

If *X1*~ *N*(μ, σ2) and *X*2 ~ *N*(μ, σ2) are two independent identically distributed random variables.

The sum of normal random variable is given by X+Y~N(μ1+ μ2, σ1^2+ σ2^2)

The Difference if normal random variable is given by X-Y~N(μ1+ μ2, σ1^2+ σ2^2)

When Z= aX the product of x is given by Z~N(aμ1,a^2σ2^2)

When Z=aX+bY, the linear combination of X and Y is given by

Z~N(aμ1 + 2^2σ^2)=2X1~N(2μ,4σ^2)

Given to find that 2X1

Thus the following the property of multiplication we get

2X~N(2μ,2^2σ^2)=2X1~N(4σ^2))

And the following the property of addition

X1+X2~N(μ+μ,σ^2+σ^2)~N(4σ^2)

And

The difference between the two is given by

2X1-(X1+X2)~N(2μ-2μ, 2σ1^2+ 4σ2^2)~N(o,6σ^2)

The mean of 2X1 and X1+X2 is same ut the var (6σ^2) of 2X1 is 2 times more than the variance of X1+X2

The difference between the two says that the two givwn variable are identically and independently distributed

1. Let X ~ N(100, 202). Find two values, *a* and*b*, symmetric about the mean, such that the probability of the random variable taking a value between them is 0.99.
2. 90.5, 105.9
3. 80.2, 119.8
4. 22, 78
5. 48.5, 151.5
6. 90.1, 109.9

Answer:

Given: p(a<x<b)=0.99,n=100,Standard Deviation = 20

From the above details

We have to exclude area of 0.005 in each of the left and right tails. Hence, we want to find the 0.5th and 99.5th percentiles Z score values

Using Python

Z value is given as stats.norm.ppf(pvalue)

Zvalue at 0.5th percentile is given as

Z(0.5) = stats.norm.ppf(0.005) = -2.576

Zvalue at 99.5 percentile is given as

Z(99.5) percentile is given as

Z(99.5)=stats.norm.ppf(0.995)=2.576

Z=(x-100)/20=>x=20z+100

a=-(20\*2.576)+100=48.5

b=(20\*2.576)+100=151.5

**D 48.5, 151.5**

Two values symmetric about mean for the given standard normal distribution are[48.5,151.5]

1. Consider a company that has two different divisions. The annual profits from the two divisions are independent and have distributions Profit1 ~ N(5, 32) and Profit2 ~ N(7, 42) respectively. Both the profits are in $ Million. Answer the following questions about the total profit of the company in Rupees. Assume that $1 = Rs. 45

Answer:

Total profit = profit 1 + profit 2 =p

Heance P~N(12,14) = (P-12)/

1. Specify a rupee range (Centered on the mean) such that it contains 95 % probability for the annual profit of the company

Answer:

Pr(P<p)=95% = Pr((P-12)/ < (P-12)/) = 0.95

Therefore P-12/~N(o,1)

I.e ǿ (P-12)/) = 0.95

Here ǿ(x)=CDF of std normal random variable

ǿ((P-12)/) = 0.95 = P - 12)/ = 1.644 = P = 12+1.6444 \* = 13.41

**$13.41 =13.41 \* 45 = RS 603.68**

1. Specify the 5th percentile of profit (in Rupees) for the company

Answer:

Assume 5th percentile profit to be z

Then pr(P<q) = 5% Pr((P-12)/ < (P-12)/) = 0.05

P-12/~N(o,1) also ǿ ((q-12)/)=0.05

From std normal distribution table

ǿ ((P-12)/)=0.05 = P-12 =-1.644

therefore P=12-1.6444 \* =10.585

**P=$10.54 = Rs 476.33**

1. Which of the two divisions has a larger probability of making a loss in a given year?

Answer:

1. For first division

Pr(of making a loss)=Pr(profit <0)

Pr(profit<0)=Pr((profit1-5)/ <(0-5)/) ~ Pr(profit1-5 < 0.883)=0.18838

1. For second division

Pr(of making a loss) = Pr(profit2<0)

Profit 2~N(7,42)= profit 7/ ~N(0,1)

Pr(profit 2<0)=Pr((profit2-7) / < -1.08012 = 0.1400

So for 1st division p(loss)=0.1888

For 2nd division p(loss) = 0.1400